Answer <u>any two</u> questions :   [2×5]

1. Find the angle through which the axes are to be rotated so that the equation $17x^2 + 18xy - 7y^2 = 1$ may be reduced to the form $Ax^2 + By^2 = 1$, $A > 0$; Find also the value of A and B.

2. If the straight lines joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and the straight line $2x + 3y + k = 0$ be at right angle, then show that $6k^2 + 5k + 52 = 0$.

3. Reduce the equation $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$ to its canonical form and hence determine the nature of the conic.

4. **Answer <u>any two</u> questions :**   [2×4]

    a) Show that the sequence $\left\{ \dfrac{n+1}{2n+1} \right\}$ is strictly monotonic decreasing and hence prove that it is convergent.

    b) State the Cauchy's Root test for convergence of a series. Use it to examine the convergence of the series $\dfrac{1}{3} + \left(\dfrac{2}{5}\right)^2 + \left(\dfrac{3}{7}\right)^3 + \left(\dfrac{4}{9}\right)^4 + ...$ [2+2]

    c) Examine the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{n^n}{n!}$.

5. **Answer <u>any one</u> question :**   [1×4]

    a) Evaluate the integral : $\displaystyle\int \dfrac{xdx}{(x-a)^2(x-b)}$.

    b) Evaluate the integral : $\displaystyle\int \dfrac{6 + 3\sin x + 14\cos x}{3 + 4\sin x + 5\cos x} dx$.

6. **Answer <u>any one</u> question :**   [1×3]

    a) Find the differential equation of all circles passing through the origin and having centres on the x-axis.

    b) Solve the differential equation $\dfrac{dy}{dx} + 1 = e^{x-y}$.

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