

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2015-18]

B.A./B.Sc. SECOND SEMESTER (January – June) 2016

Mid-Semester Examination, March 2016

Date : 21/03/2016

MATHEMATICS (General)

Time : 12 noon – 1 pm

Paper : II

Full Marks : 25

Answer any two questions :

[2×5]

1. Find the angle through which the axes are to be rotated so that the equation $17x^2 + 18xy - 7y^2 = 1$ may be reduced to the form $Ax^2 + By^2 = 1$, $A > 0$; Find also the value of A and B.
2. If the straight lines joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and the straight line $2x + 3y + k = 0$ be at right angle, then show that $6k^2 + 5k + 52 = 0$.
3. Reduce the equation $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$ to its canonical form and hence determine the nature of the conic.

4. Answer any two questions :

[2×4]

- a) Show that the sequence $\left\{ \frac{n+1}{2n+1} \right\}$ is strictly monotonic decreasing and hence prove that it is convergent.

- b) State the Cauchy's Root test for convergence of a series. Use it to examine the convergence of the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \left(\frac{4}{9}\right)^4 + \dots$.

[2+2]

- c) Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

5. Answer any one question :

[1×4]

- a) Evaluate the integral : $\int \frac{x dx}{(x-a)^2(x-b)}$.
- b) Evaluate the integral : $\int \frac{6+3\sin x + 14\cos x}{3+4\sin x + 5\cos x} dx$.

6. Answer any one question :

[1×3]

- a) Find the differential equation of all circles passing through the origin and having centres on the x-axis.
- b) Solve the differential equation $\frac{dy}{dx} + 1 = e^{x-y}$.

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